By substituting the values of $x$ found above in the original equation, we see that 0 and $\pm\ \infty$ are to be rejected. We find, however, by inspection that $x = 1$ is a root.

Hence, the roots are 1 and $(1 \pm \sqrt{5})/2$.

Also solved by ALBERT N. NAUER, A. M. HARDING, C. E. GITHENS, V. M. SPUNAR, ELIJAH SWIFT, W. C. EELLS, G. W. HARTWELL, and the PROPOSER.

**430A. Proposed by H. C. FEEMSTER, York College, Neb.**

Solve the equations

$$\sum_{i=1}^{n} x_i - x_n \quad = k + \frac{n^2 - 3n + 2}{2} d, \tag{1}$$

$$\sum_{i=1}^{n} x_i - x_{n-1} = k + \frac{n^2 - 3n + 4}{2} d, \tag{2}$$

$$\sum_{i=1}^{n} x_i - x_{n-2} = k + \frac{n^2 - 3n + 6}{2} d, \tag{3}$$

$$\cdots\cdots\cdots\cdots\cdots \qquad \vdots$$

$$\sum_{i=1}^{n} x_i - x_1 \quad = k + \frac{n^2 - n}{2} d. \tag{n}$$

SOLUTION BY A. M. HARDING, Univ. of Arkansas.

Add the given equations and obtain

$$(n-1)\sum_{i=1}^{n} x_i = nk + \frac{n^3 - 3n^2 + n^2 + n}{2} d,$$

or

$$\sum_{i=1}^{n} x_i = \frac{n}{n-1} \cdot k + \frac{n(n-1)}{2} d.$$

Subtract each of the given equations from this equation and obtain

$$x_n = \frac{k}{n-1} + (n-1)d,$$

$$x_{n-1} = \frac{k}{n-1} + (n-2)d,$$

$$x_{n-2} = \frac{k}{n-1} + (n-3)d,$$

$$\cdots\cdots\cdots\cdots$$

$$x_2 = \frac{k}{n-1} + d,$$

$$x_1 = \frac{k}{n-1}.$$

Also solved by NATHAN ALTSHILLER, S. A. JOFFE, J. W. CLAWSON, FRANK R. MORRIS, ELBERT H. CLARKE, HORACE OLSON, N. P. PANDYA, and the PROPOSER.

**431. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.**

Form a magic square of 9 cells such that (the integers being all different) the products of the integers in the rows, columns, and diagonals shall be the same and the smallest product possible.

SOLUTION BY S. A. JOFFE, New York City.

All the nine integers will be determined if, in addition to the product $p$ of three integers of any row (column, or diagonal), we know two extremes, say of the first row, $a$ and $b$, and the central integer $c$.

| | | |
|---|---|---|
| $a$ | | $b$ |
| | $c$ | |
| | | |

The extremes of the third row will then be $p/ac$ and $p/bc$; of the second row will be $bc/a$ and $ac/b$; and those of the second column will be $p/ab$ and $ab/c$.

Hence the product of the integers in the second row equals

$$\frac{bc}{a} \cdot c \cdot \frac{ac}{b} = c^3,$$

and therefore

$$p = c^3,$$

so that the magic square becomes

| | | |
|---|---|---|
| $a$ | $\frac{c^3}{ab}$ | $b$ |
| $\frac{bc}{a}$ | $c$ | $\frac{ac}{b}$ |
| $\frac{c^2}{b}$ | $\frac{ab}{c}$ | $\frac{c^2}{a}$ |

If we disregard the central integer $c$, the remaining eight integers must therefore form four pairs: (1) $m_1$ and $c^2/m_1$; (2) $m_2$ and $c^2/m_2$; (3) $m_3$ and $c^2/m_3$; (4) $m_4$ and $c^2/m_4$. Taking now for $m_1$, $m_2$, $m_3$ and $m_4$ the smallest four integers, *i. e., 1, 2, 3 and 4*, we find that the *smallest square* number $c^2$, divisible by these four integers, is *36*, and consequently $c = \sqrt{36} = 6$.

However, no corner number, say $a$, can be unity; because, if $a = 1$ then $ac/b$ and $ab/c$ become $c/b$ and $b/c$, which must be simultaneously integers, and this is impossible unless $b = c$; but the latter equality is excluded by the condition of the problem. Taking $a = 2$, $b = 3$ and $c = 6$, we find the required magic square with the smallest product possible to be

| | | |
|---|---|---|
| 2 | 36 | 3 |
| 9 | 6 | 4 |
| 12 | 1 | 18 |

Also solved by HERBERT N. CARLETON, N. P. PANDYA.